In-Class Problem
Monopoly Market Structure

In this problem we're going to take a look at the differences between monopoly and competitive goods market structures for the same good. We'll consider social welfare, consumer surplus, profit, output and price for each market form. Let's assume that the producer is a profit maximizer, that the market for the factor inputs in production of this good is perfectly competitive, and that this good's market is parameterized by the following:

Demand: $P = 100 - 4Q_D$ (1)
Marginal Revenue: $P = 100 - 8Q_D$ (2)
Costs: $MC = AC = \$20$ (3)

$MC = MR = P^*$ (4)

Which happens to be \$20 in this case. To find Q* we simply substitute (3) into (1)

$\$20 = 100 - 4Q_D$ (5)
$4Q_D = 80$
$Q^* = 20$

Notice that the demand and marginal revenue equations are in the "inverse" form, so let's convert them each to the direct form by normalizing each on Q_D:

Demand:
$$P = 100 - 4Q_D$$
$$4Q_D = 100 - P$$
$$Q_D = 25 - \tfrac{1}{4}P \qquad (6)$$

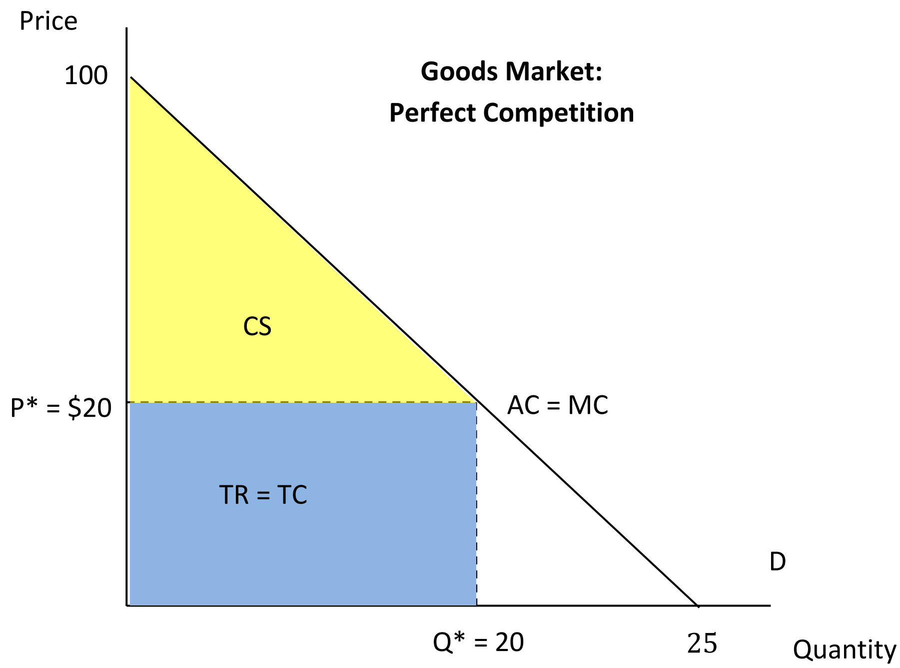
Marginal Revenue:
$$P = 100 - 8Q_D$$
$$8Q_D = 100 - P$$
$$Q_D = 12\tfrac{1}{2} - \tfrac{1}{8}P \qquad (7)$$

Now calculate the Demand and Marginal Revenue curve intercepts by setting the variables equal to 0:

Demand: if $P = 0$, then $Q_D = 25 - \frac{1}{4}(0) = 25$
if $Q_D = 0$, then $0 = 25 - \frac{1}{4}P$
$\frac{1}{4}P = 25$ and $P = 100$

Marginal Revenue: if $P = 0$, then $Q_D = 12\frac{1}{2} - \frac{1}{8}(0) = 12\frac{1}{2}$
if $Q_D = 0$, then $0 = 12\frac{1}{2} - \frac{1}{8}P$
$\frac{1}{8}P = 12\frac{1}{2}$ and $P = 100$

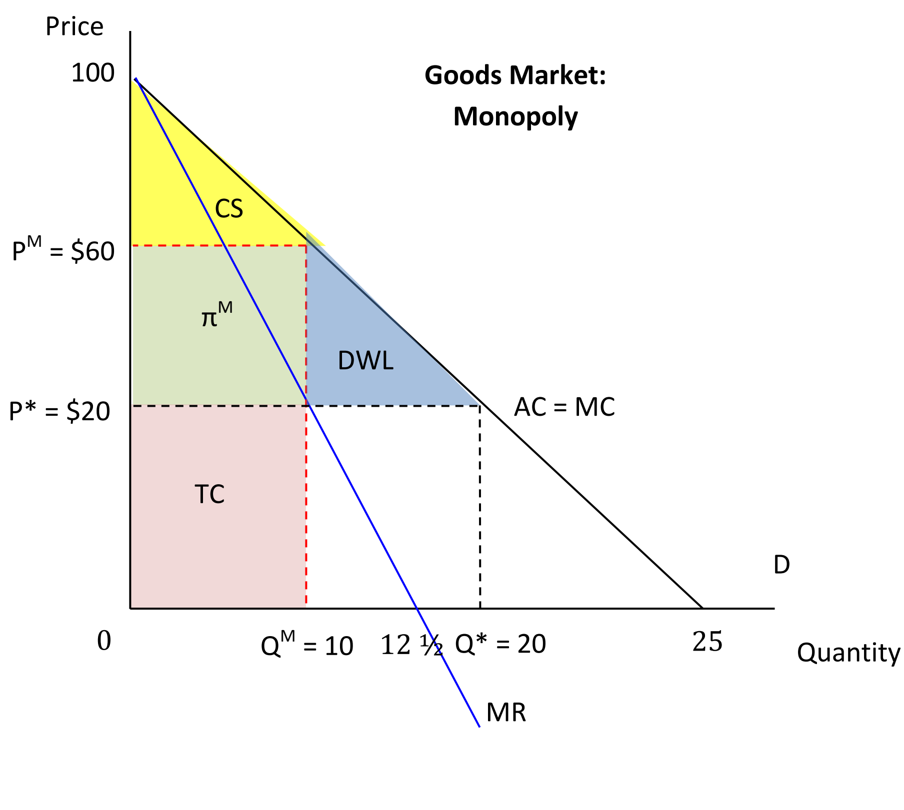
With that, let's take a look at the goods market model under the condition of perfect competition:



$$CS^C = \frac{(20)(100-20)}{2} = \$800 \tag{8}$$

$$\pi^C = \$0;\ \ DWL = \$0 \tag{9}$$

Now let's take a look at this goods market under the condition of monopoly:



To calculate QM we simply substitute $P^* = 20$ in the Marginal Revenue equation

$$Q_D = 12\tfrac{1}{2} - \tfrac{1}{8}(20) = 10 = Q^M \tag{10}$$

Now that we know Q^M, we can identify P^M by substituting Q^M for Q_D into the Demand equation

$$10 = 25 - \tfrac{1}{4}P$$
$$\tfrac{1}{4}P = 15$$
$$P^M = \$60 \tag{11}$$

I've noted the areas representing CS^M, TC, π^M (profit), and DWL in this monopoly model so now we can calculate these as well as TR^M with the values given:

$$CS^M = \frac{(10)(100-60)}{2} = \$200 \tag{12}$$

$$TC = (\$20 - 0)(10) = \$200 \tag{13}$$

$$\pi^M = (10)(\$60 - \$20) = \$400 \tag{14}$$

$$TR^M = TC + \pi = \$200 + \$400 = \$600 \tag{15}$$

$$DWL = \frac{(20-10)(60-20)}{2} = \$200 \tag{16}$$

Now let's turn our attention to a social welfare analysis:

$$W^C = CS^C + \pi^C = \$800 + \$0 = \$800 \tag{18}$$

$$W^M = CS^M + \pi^M = \$200 + \$400 = \$600 \tag{19}$$

$$W^C - W^M = \$800 - \$600 = DWL \tag{20}$$