Public Good Problem: Robinson and Friday[1]
In-Class Problem[2]

Consider a society inclusive of two agents, Robinson and Friday, in which there is one public good: a banana tree. We know the tree is a public good because it produces more bananas than the agents can consume and there is never shortage of bananas, but it does require some pruning and care to produce the quality of banana the agents enjoy. Let's assume that there is no currency in this society and that the price each agent might have to pay is some unit(s) of labor, which we'll call P^i and that the preferred/optimal quantity of bananas is Q*. We take as given that the marginal cost of a banana in this society is equal to 10 (MC=10).

Also, recall the governing rules of Public Goods:
$Q^E = Q^K = Q^J = Q$
$P = P^E + P^K + P^J$
P = MC

The following represent each agent's demand for bananas:

Robinson Demand^{Robinson}: $Q^R = 200 – 15\ P^R$ (1)

Friday Demand^{Friday}: $Q^F = 150 – 35\ P^F$ (2)

With this information, answer the following questions:

a. What is the optimal quantity (Q) of bananas in this society?

To find this value we first need to convert the demand equations in to inverse demand equations normalized on P, this way we can sum P^R and P^F to get P for the public goods market

$$Q^R = 200 - 15\,P^R$$
$$15\,P^R = 200 - Q^R$$
$$P^R = \frac{200}{15} - \frac{1}{15}Q^R$$
$$P^R = 13.33 - .067Q^R \quad (3)$$

$$Q^F = 150 - 35\,P^F$$
$$35\,P^F = 150 - Q^F$$
$$P^F = \frac{150}{35} - \frac{1}{35}Q^F$$
$$P^F = 4.29 - .0286Q^F \quad (4)$$

Sum (3) and (4)

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[1] This primer is intended to present an abbreviated discussion of the included economic concepts and is not intended to be a full or complete representation of them or the underlying economic foundations from which they are built.

[2] This problem set was developed by Rick Haskell (rick.haskell@utah.edu), Ph.D. Student, Department of Economics, College of Social and Behavioral Sciences, The University of Utah, Salt Lake City, Utah (2014).

$$P^R + P^F = 13.33 - .067Q^R + 4.29 - .0286Q^F \quad (5)$$

Since $P^R + P^F = P$ and $Q^R = Q^F = Q$ we can rewrite (5) as

$$P = 17.62 - .0956Q \quad (6)$$

Since MC = P we can substitute 10 for P and solve for Q

$$10 = 17.62 - .0956Q$$

$$.0956Q = 17.62 - 10$$

$$Q^* = \frac{7.62}{.0956} = 79.70 \quad (7)$$

b. How much are Robinson and Friday each prepared to pay to consume bananas?

To calculate these values we can substitute Q* for QF and QR and solve for P (we could also take the inverse demand equations and solve for the same values, either way we'll get the same outcomes).

$$Q^R = 200 - 15\,P^R$$
$$79.70 = 200 - 15\,P^R$$
$$15\,P^R = 120.3$$
$$P^R = \frac{120.3}{15} = 8.02 \quad (8)$$

$$Q^F = 150 - 35\,P^F$$
$$79.70 = 150 - 35\,P^F$$
$$35\,P^F = 70.30$$
$$P^F = \frac{70.30}{35} = 2.00 \quad (9)$$

c. What is the total price this society is prepared to pay to consume bananas?

$P^R + P^F = 8.02 + 2.00 = 10.02$ which is roughly equal to 10, which was the P = MC value

d. What does this tell us about the Robinson and Friday's relative preferences in respect to the consumption of bananas?

Given that Robinson's demand results in a higher P than does Friday's, it suggests that Robinson values bananas more highly than does Friday – his preference towards them is greater than Friday's.