Time Value of Money
Simple Equation Problem Set
BUSI 101B
Fall 2016

Understanding the dynamics of time and money is critical to the mastery both economics and finance. It is so woven into the fabric of Corporate Finance that understanding how to adequately assign value is virtually impossible without such a knowledge.   In this problem set we discuss using algebraic equations and financial calculators to solve for one of the five critical variables in time value of money calculations. These variables, in a very general sense, are Present Value, Future Value, Cash Flow, rate, and time. The basic equational form we'll use is $PV = \sum \frac{\beta^t}{(1+r)^t}$, in which the *t* subscript is simply a descriptor, while the superscript *t* is an exponential operator. The variable β represents some flow of funds, or cash flow, whether in or out.

a.   You've been offered a $1,000 investment with a promised payout of $10,000 in 20 years. If you were to participate in the investment, what would be your annual rate of return? *Solve this problem using Time Value of Money equations and be sure to show all of your work. Express your answer as a percentage rounded to the nearest one hundredth*

   r = .1220 or 12.20%

$$r = \sqrt[t]{\frac{\beta_t}{PV}} - 1$$

b.   One of your roommates has come to you in need of as much cash as possible. The only thing they have to offer is an expected cash payout of $1,800 from a trust in four years. You're willing to loan the money at 6% interest. How much would you be prepared to loan the roommate? *Solve this problem using Time Value of Money equations and be sure to show all of your work. Use basic currency notation to express your answer*

   PV = $1,425.77

$$PV = \frac{\beta_t}{(1+r)^t}$$

c.   How long might one expect an investment of $800 to pay out at $1,200 if the investment earned a 6% rate of return? *Solve this problem using Time Value of Money equations and be sure to show all of your work. Express your answer in years rounded to the nearest one hundredth.*

   t = 6.96

$$t = \frac{\ln\left(\frac{\beta_t}{PV}\right)}{\ln(1 + r)}$$

Suppose your broker has come to you with an investment opportunity expected to payout in 5 years at a 10% rate of return. If you placed $15,000 into this investment, how much would you expect to receive at the end of the 5 year period? *Solve this problem using Time Value of Money equations and be sure to show all of your work. Use basic currency notation to express your answer*

FV = β_t = $24,157.65

$$FV = PV(1 + r)^t$$

d. Suppose you want to determine the present value of a stream of income from an investment asset you expect will offer you a $1,500 annual income, over a period of 5 years, based on the promise of a 6% rate of return, and for which there is no salvage value at the end of the 5 year term. How much would you be prepared to offer for such an investment? Show both equational and calculator solutions to this problem.

Manual calculation: this is a constant cash flow equation

$$PV = \sum \frac{\beta^t}{(1+r)^t} \qquad (1)$$

$$= \frac{1,500}{1.06^1} + \frac{1,500}{1.06^2} + \frac{1,500}{1.06^3} + \frac{1,500}{1.06^4} + \frac{1,500}{1.06^5}$$

$$= 1415.09 + 1334.99 + 1259.43 + 1188.14 + 1120.89$$

$$= 6318.55$$

Financial Calculator

Make sure P/YR is set to 1 to reflect annual cash flow and that the calculator is set to calculate values as of the end of each period. Then input the following values:

- I/YR = 6
- PMT = 1500
- N = 5
- FV = 0

And solve for PV = -6318.55. Notice that the calculator returned a negative value. This is to recognize that you would need to pay this amount for the investment. Negative values reflect cash flows out, while positive values reflect cash flows in.

e. Now suppose you're considering purchasing a car using a loan you'll arrange through the local dealership. In this case you don't plan to put any money down and you've explained you have a budget of no more than $500 per month. The sales person shows you a used BMW 325i for that amount, you sign a few pieces of paper, and off you go, happy with your new car – nice car BTW. How do you determine how much you've agreed to pay for the car?

The amount paid is a function of four of the five variables in a time value of money calculation. In this case you know the amount of the payment and presume that the future value is $0 (which is relatively standard among installment loans), but you don't know the rate or term of the loan, or the amount of the loan. With the information provided you can't tell how much you've agreed to pay for the car. Which, as it turns out, is why the sales person doesn't focus on that detail until you're signing the documents and have already emotionally committed to the transaction.

f. Suppose the loan is for 72 months (6 years) at 6% interest. How much is the amount of the loan – and in this case, the price of the car? Use your calculator to solve this problem and indicate the values you entered for each of the required variables.

Financial Calculator

Make sure P/YR is set to 12 to reflect annual cash flow and that the calculator is set to calculate values as of the end of each period. Then input the following values:

- I/YR = 6
- PMT = -500
- N = 72
- FV = 0

Solve for PV = 30,169.76. Notice here that the sign of the payment amount you input is negative, but the calculation returns with a positive value. That's because the lender is going to give you the funds to purchase the car (money in) based on your promise to pay $500 per month (money out).