Time Value of Money[1]
In-Class Problem[2]

Understanding the dynamics of time and money is critical to the mastery both economics and finance. It is so woven into the fabric of Corporate Finance that understanding how to adequately assign value is virtually impossible without such a knowledge.

In this problem set we discuss using algebraic equations and financial calculators to solve for one of the five critical variables in time value of money calculations. These variables, in a very general sense, are Present Value, Future Value, Cash Flow, rate, and time.

The basic equational form we'll use is $PV = \sum \frac{\beta^t}{(1+r)^t}$, in which the *t* subscript is simply a descriptor, while the superscript *t* is an exponential operator. The variable β represents some flow of funds, in or out.

a. Suppose you want to determine the present value of a stream of income from an investment asset you expect will offer you a $1,500 annual income, over a period of 5 years, based on the promise of a 6% rate of return, and for which there is no salvage value at the end of the 5 year term. How much would you be prepared to offer for such an investment? Show both equational and calculator solutions to this problem.

Manual calculation: this is a constant cash flow equation

$$PV = \sum \frac{\beta^t}{(1+r)^t} \quad \textbf{(1)}$$

$$= \frac{1{,}500}{1.06^1} + \frac{1{,}500}{1.06^2} + \frac{1{,}500}{1.06^3} + \frac{1{,}500}{1.06^4} + \frac{1{,}500}{1.06^5}$$

= 1415.09 + 1334.99 + 1259.43 + 1188.14 + 1120.89

= 6318.55

Financial Calculator

Make sure P/YR is set to 1 to reflect annual cash flow and that the calculator is set to calculate values as of the end of each period. Then input the following values:

- I/YR = 6
- PMT = 1500
- N = 5
- FV = 0

[1] This problem and solution set is intended to present an abbreviated discussion of the included finance concepts and is not intended to be a full or complete representation of them or the underlying foundations from which they are built.

[2] This problem set was developed by Richard Haskell, PhD (rhaskell@westminstercollege.edu), Gore School of Business, Westminster College, Salt Lake City, Utah (2015).

And solve for PV = -6318.55. Notice that the calculator returned a negative value. This is to recognize that you would need to pay this amount for the investment. Negative values reflect cash flows out, while positive values reflect cash flows in.

b. How would PV change if this investment is expected to have a salvage value of $5,000 at the end of the fifth year? Show both equational and calculator solutions to this problem.

Manual calculation: this is a constant cash flow equation for all but the final year

$$\boldsymbol{PV} = \sum \frac{\boldsymbol{\beta^t}}{(\mathbf{1}+\boldsymbol{r})^t} \qquad \textbf{(1)}$$

$$= \frac{1{,}500}{1.06^1} + \frac{1{,}500}{1.06^2} + \frac{1{,}500}{1.06^3} + \frac{1{,}500}{1.06^4} + \frac{6{,}500}{1.06^5}$$

$$= 1415.09 + 1334.99 + 1259.43 + 1188.14 + 4857.18$$

$$= 10054.84$$

Financial Calculator

Make sure P/YR is set to 1 to reflect annual cash flow and that the calculator is set to calculate values as of the end of each period. Then input the following values:

- I/YR = 6
- PMT = 1500
- N = 5
- FV = 5000

And solve for PV = -10054.84. In this case FV has changed from 0 to 5000. Notice that the value didn't increase by 5000, but increased by the time discounted value of $5000 to be received five years into the future.

c. Now suppose you're considering purchasing a car using a loan you'll arrange through the local dealership. In this case you don't plan to put any money down and you've explained you have a budget of no more than $500 per month. The sales person shows you a used BMW 325i for that amount, you sign a few pieces of paper, and off you go, happy with your new car – nice car BTW. How do you determine how much you've agreed to pay for the car?

The amount paid is a function of four of the five variables in a time value of money calculation. In this case you know the amount of the payment and presume that the future value is $0 (which is relatively standard among installment loans), but you don't know the rate or term of the loan, or the amount of the loan. With the information provided you can't tell how much you've agreed to pay for the car. Which, as it turns out, is why the sales person doesn't focus on that detail until you're signing the documents and have already emotionally committed to the transaction.

d. Suppose the loan is for 72 months (6 years) at 6% interest. How much is the amount of the loan – and in this case the price of the car? Show the equational form (not the entire equation) and calculator solution to this problem.

Equational form: this is a constant cash flow equation

$$\boldsymbol{PV} = \sum \frac{\boldsymbol{\beta^t}}{(\mathbf{1}+\boldsymbol{r})^t}$$
$$= \frac{500}{1.005^1} + \cdots + \frac{500}{1.005^{72}}$$
$$= 30{,}169.76$$

Notice that the equation includes negative values for each of the cash flows (payments). This indicates that these are payments being made by you (money going away from you). That the sign if the PV is negative is simply reflective of this.

Financial Calculator

Make sure P/YR is set to 12 to reflect annual cash flow and that the calculator is set to calculate values as of the end of each period. Then input the following values:

- I/YR = 6
- PMT = -500
- N = 72
- FV = 0

Solve for PV = 30,169.76. Notice here that the sign of the payment amount you input is negative, but the calculation returns with a positive value. That's because the lender is going to give you the funds to purchase the car (money in) based on your promise to pay $500 per month (money out).

e. After thinking about this for a bit you decide that $500 is just too much for you to afford each month. When you tell the sales person you're surprised, you're told "no problem, we can reduce your payment to $450 and your interest rate and price won't change." If this is the case, what has changed and what are the details of the change. You can do this with your calculator and don't need to show or solve equations.

If the interest rate and price are the same, and we'll assume the future value to remain at $0, but the payment has gone down to $450, then the only variable unaccounted for is time. Input the following values into your calculator and solve for N (the time variable).

- I/YR = 6
- PMT = -450
- PV = 30,169.76
- FV = 0

Solve for N = 81.86 months – or almost 82 months. So a monthly payment reduction of 10% increased the length of your loan by almost 14%.

f. Finally, let's assume you've been offered a business opportunity with an up-front capital cost of $250,000, which business is expected to need annual capital infusions of $10,000 per year for the first three years, and will run at break even for two additional years before it begins to return a positive cash flow of $50,000 a year for the following five years. You believe you can sell this business for $350,000 at the end of 10 years and your opportunity cost of capital is 6%. Is this business opportunity worth the price? Show both equational and calculator solutions to this problem and explain the solution you've calculated.

Manual calculation: this is a non-constant cash flow equation

$$\boldsymbol{PV} = \sum \frac{\boldsymbol{CF^t}}{(\mathbf{1}+\boldsymbol{r})^t}$$

$$= \frac{-250{,}000}{1.06^0} + \frac{-10{,}000}{1.06^1} + \frac{-10{,}000}{1.06^2} + \frac{-10{,}000}{1.06^3} + \frac{0}{1.06^4} + \frac{0}{1.06^5} + \frac{50{,}000}{1.06^6} + \frac{50{,}000}{1.06^7} + \frac{50{,}000}{1.06^8} + \frac{50{,}000}{1.06^9} + \frac{400{,}000}{1.06^{10}}$$

= -250,000 – 9433.96 – 8899.96 – 8,396.19 + 0 + 0 + 35,248.03 + 33252.86 + 31,370.62 + 29,594.92 + 223,357.91

= 76,094.22

Financial Calculator:

- Be sure to set P/YR to 1 to reflect annual cash flow amounts
- I/YR = 6
- You don't need to set the value of N or FV as the CF register function interprets these values based on your CF_i entries.

CF_0	CF_1	CF_2	CF_3	CF_4	CF_5	CF_6	CF_7	CF_8	CF_9	CF_{10}
-250,000	-10,000	-10,000	-10,000	0	0	50,000	50,000	50,000	50,000	400,000

Solve for NPV = $76,094.22

Okay, what does the $76,094.22 value the equation and calculator both return mean? In this case we've endogenized the costs and benefits into the equations/calculations, so the computed value is the net present value. This needs to be compared to a value of 0. If NPV > 0, then the investment makes sense. If NPV < 0, the investment doesn't make sense. If NPV = 0, then you're fundamentally indifferent and could take it or leave it.