Valuation Framework: APV[1]
In-Class Problem[2]

As you continue to consider various valuation metrics, you've decided to expand your analysis to include Adjusted Present Value (APV) as a particular type of Discounted Cash Flow (DCF) valuation framework. You understand that APV highlights value changes resulting from changing capital structure more easily than do $\text{Value}_{DCF/KVD}$ or $\text{Value}_{DCF/DG}$ models using weighted average cost of capital (WACC) as the discount rate, and that DCF of a firm's free cash flow (FCF) explicitly highlights when a company creates value net of its capital expenditures. All of which is important to you and the private equity group with which you're working.

In this problem set we're going to assume the following:

- **The firm is capitalized by bonds with a face value of \$25,000 and coupon rate of 6%, Common Stock with a value of \$200,000, and retained earnings of \$75,000**
- **NOPLAT_{2015} = \$15,000.00; ROIC = 15.33% (kind of a specific figure, you'll see why later on)**
- **Growth of NOPLAT from 2015-2025 = $g_{2015\text{-}2025}$ = 8%**
- **Growth of NOPLAT and FCF from 2026 onward = g_{2026} = 4%**
- **$\text{Depreciation}_{2015}$ = 1,200 and is expected to remain constant for the foreseeable future**
- **$\text{Invested Capital}_{2014}$ = 93,587.25**
- **The firm's Net Investment is expect to remain constant for the foreseeable future**
- **Interest_{2015} = \$1,000 and expands at the same rate as NOPLAT**
- **The cost of debt (k_d) = 6%; the unlevered cost of equity and tax (k_u and k_{tax}) each equal = 12%**
- **The firm's average tax rate is constant at 35%.**

The APV calculation separates the firm's value into two components: the discounted free cash flow at the unlevered cost of capital, plus the discounted tax shield at the unlevered cost of equity or the cost of tax. We know that APV = V_{FCF} + V_{TAX} which can be broken into its component parts such that

- $V_{FCF} = PV_{DCF(FCF)} + PV_{CV(FCF)}$
  - **where** $PV_{DCF(FCF)} = \sum_{t=1}^{\infty} \frac{FCF}{(1+k_u)^t}$; $PV_{CV(FCF)} = \frac{\frac{FCF}{(k_u - g)}}{(1+k_u)^t}$
- $V_{TAX} = PV_{DCF(TAX)} + PV_{CV(TAX)}$
  - **where** $PV_{DCF(TAX)} = \sum_{t=1}^{\infty} \frac{(T_m)\text{Interest}}{(1+k_{tax})^t}$; $PV_{CV(TAX)} = \frac{\frac{Tax\ Shield_1}{(k_{tax} - g)}}{(1= k_{tax})^t}$

Note that for the continuing value we're using an income augmented form of the Dividend Growth equation

Note that we're using an income augmented form of the Dividend Growth equation for the Continuing Value. To calculate our values we'll work off of the value and rate assumptions above with some of the following definitions:

- **Recall that k_u is unobservable and we need to make an assumption regarding its value, which we'll assume at $k_u = k_{tax}$**
  - **This is true with a constant D/E ratio, which we'll also assume**

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[1] This problem and solution set is intended to present an abbreviated discussion of the included finance concepts and is not intended to be a full or complete representation of them or the underlying foundations from which they are built.
[2] This problem set was developed by Richard Haskell, PhD (rhaskell@westminstercollege.edu), Gore School of Business, Westminster College, Salt Lake City, Utah (2015).

- **k_u = unlevered cost of equity**
- **k_{tax} = cost of capital for the tax shields**
- **T_m is the marginal corporate tax rate**
- **k_d = cost of debt**

Finally, the APV framework also leads to the formation of k_e, or the levered cost of equity, through the Modigliani and Miller theorem: $k_e = k_u + \frac{D}{E}(k_u - k_d) - \frac{V_{TAX}}{E}(k_u - k_{TAX})$. In this case $\frac{V_{TAX}}{E}(k_u - k_{TAX}) = 0$ because $k_u = k_{TAX}$ and $k_u - k_{TAX} = 0$, so the term is equal to 0 and in this case we can simply note that $k_e = k_u + \frac{D}{E}(k_u - k_d)$. This is the result of our assumption that $\Delta\frac{D}{E} = 0$

a. List the annual free cash flow (FCF) values for the 10 year period 2016-2025.

Recall that FCF = NOPLAT + Depreciation – Net Investment; $NOPLAT_{2015} \times (1 + g_{2015\text{-}2025}) = NOPLAT_{2016}$ and that $\text{Invested Capital}_{t+1} - \text{Invested Capital}_t + \text{Depreciation} = \text{Net Investment}$

Year	NOPLAT	Dep	Net Investment	FCF
2016	16,200.00	1,200	4,200	13,200.00
2017	17,496.00	1,200	4,200	14,496.00
2018	18,895.68	1,200	4,200	15,895.68
2019	20,407.33	1,200	4,200	17,407.33
2020	22,039.92	1,200	4,200	19,039.92
2021	23,803.11	1,200	4,200	20,803.11
2022	25,707.36	1,200	4,200	22,707.36
2023	27,763.95	1,200	4,200	24,763.95
2024	29,985.07	1,200	4,200	26,985.07
2025	32,383.87	1,200	4,200	29,383.87

b. Calculate $PV_{DCF(FCF)}$, $PV_{CV(FCF)}$ and provide the value of V_{FCF}. *Be sure to show all of your work*.

Recall $V_{FCF} = PV_{DCF(FCF)} + PV_{CV(FCF)}$. Start with the equation $PV_{DCF(FCF)} = \sum_{t=1}^{\infty}\frac{FCF}{(1+k_u)^t}$, which is simply a DCF equation using non-constant cash flows through your HP10bii, and should result in a value of 106,527.32. Be sure you have P/YR = 1 and recall that k_u = 12%.

Year	FCF	PV_{FCF}	Total PV_{FCF}
2016	13,200.00	11785.71	11785.71
2017	14,496.00	11556.12	23341.84
2018	15,895.68	11314.23	34656.07
2019	17,407.33	11062.68	45718.74
2020	19,039.92	10803.76	56522.51
2021	20,803.11	10539.51	67062.01
2022	22,707.36	10271.66	77333.67
2023	24,763.95	10001.75	87335.42
2024	26,985.07	9731.09	97066.50
2025	29,383.87	9460.82	106527.32

We can validate these values by doing the math ourselves through the following equations:

$$PV_{DCF/FCF} = \sum_{t=1}^{\infty} \frac{\text{FCF}}{(1+k_u)^t} = \frac{13200}{(1.12)^1} + \frac{14496}{(1.12)^2} + \frac{15895.68}{(1.12)^3} + \frac{17407.33}{(1.12)^4} + \frac{19039.92}{(1.12)^5} + \frac{20803.11}{(1.12)^6} + \frac{22707.36}{(1.12)^7} + \frac{24763.95}{(1.12)^8} + \frac{26985.07}{(1.12)^9} + \frac{29383.87}{(1.12)^{10}}$$

= 11785.71 + 11556.12 + 11314.23 + 11062.68 + 10803.76 + 10539.51 + 10271.66 + 10001.75 + 9731.09 + 9460.82

= 106,527.32

To this value we need to add $PV_{CV(FCF)} = \frac{\frac{FCF_1}{(k_u - g)}}{(1+k_u)^t}$. Recall that $FCF_1 = FCF_{2026} = NOPLAT_{2025}$ x (1.04) – Net Investment = 33,679.23 – 3,000 = 30,679.23.

So $PV_{CV(FCF)} = \frac{\frac{30{,}559.23}{(.12-.04)}}{(1.12)^{10}} = \frac{381{,}990.37}{(1.12)^{10}}$ = 122,990.68 and V_{FCF} = 106,527.32 + 122,990.68 = 229,518.00

c. **Calculate and list the interest tax shield values for the 10 year period 2016-2025**

Recall that the interest tax shield = Interest_i x $T_{AVERAGE}$

	Interest	Average Tax Rate	Interest Tax Shield
2016	1,080.00	35%	378.00
2017	1,166.40	35%	408.24
2018	1,259.71	35%	440.90
2019	1,360.49	35%	476.17
2020	1,469.33	35%	514.26
2021	1,586.87	35%	555.41
2022	1,713.82	35%	599.84
2023	1,850.93	35%	647.83
2024	1,999.00	35%	699.65
2025	2,158.92	35%	755.62

d. Calculate $PV_{DCF(TAX)}$, $PV_{CV(TAX)}$ and provide the value of V_{TAX}. *Be sure to show all of your work***.**

Recall that $k_{tax} = k_u$ = 12% in this scenario and $V_{FCF} = PV_{DCF(FCF)} + PV_{CV(FCF)}$. Start with the equation Start with $PV_{DCF(TAX)} = \sum_{t=1}^{\infty} \frac{(T_m)\text{Interest}}{(1+k_{tax})^t}$

	Interest Tax Shield	$PV_{Tax\ Shield}$	Total $PV_{Tax\ Shield}$
2016	378.00	337.50	**337.50**
2017	408.24	325.45	**662.95**
2018	440.90	313.82	**976.77**
2019	476.17	302.62	**1279.39**
2020	514.26	291.81	**1571.19**
2021	555.41	281.39	**1852.58**
2022	599.84	271.34	**2123.92**
2023	647.83	261.65	**2385.56**
2024	699.65	252.30	**2637.86**
2025	755.62	243.29	**2881.15**

$$PV_{DCF/TAX} = \sum_{t=1}^{\infty} \frac{\text{Interest x } T_M}{(1+k_{tax})^t} = \frac{\text{Interest Tax Shield}}{(1+k_{tax})^t} = \frac{378}{(1.12)^1} + \frac{408.24}{(1.12)^2} + \frac{440.90}{(1.12)^3} + \frac{476.17}{(1.12)^4} + \frac{514.26}{(1.12)^5} + \frac{555.41}{(1.12)^6} + \frac{599.84}{(1.12)^7} + \frac{647.83}{(1.12)^8} + \frac{699.65}{(1.12)^9} + \frac{755.62}{(1.12)^{10}}$$

= 2881.15

To this value we need to add $PV_{CV(TAX)} = \frac{\frac{Interest\ Tax\ Shield_1}{(k_{tax} - g)}}{(1 + k_{tax})^t}$.

Recall that Interest Tax Sheild$_1$ = Interest Tax Shield$_{2026}$ = Interest$_{2026}$ x T_M = 2,245.28 x .35 = 785.85

So $PV_{CV(TAX)} = \frac{\frac{785.85}{(.12 - .04)}}{(1\ .12)^{10}} = \frac{9{,}823.11}{(1\ .12)^{10}}$ = 3,162.78 and V_{TAX} = 2,881.15 + 3,162.78 = 6,043.93

e. What is VALUE$_{APV}$ for the subject firm?

We can sum these as APV = V_{FCF} + V_{TAX} = 229,518.00 + 6,043.93 = 235,561.93

f. How would you describe the valuation calculated using the VALUE$_{APV}$ model versus a VALUE$_{DCF(KVD)}$ or VALUE$_{DCF(DG)}$? What is its relevance to investors?

APV highlights value changes as a result of changing cash flows and changes in a firm's capital structure. It gives the firm and its investors a view of the firm's value from a different perspective that does VALUE$_{KVD}$ which leaves capital structure changes out of the equation, even though it MIGHT be argued that such changes have potential impact on the firm's cash flows.

g. What is the value for the blended cost of equity (k_e) given the values derived for k_u and k_{tax}, and the firm's debt to equity ratio $\left(\frac{D}{E}\right)$?

Recall that a firm's equity from a capital perspective is equal to the value of its long-term debt plus its owner's equity. In this case D = Debt (bonds, mortgages, credit lines, etc) = 25,000, and E = Common Stock + Preferred Stock + Retained Earnings = 200,000 + 75,000 = 275,000

To calculate k_e we'll rely on the Modigliani & Miller theorem equation, $k_e = k_u + \frac{D}{E}(k_u - k_d)$. We have values for k_u and k_d and need to calculate the debt to equity ratio $\left(\frac{D}{E}\right) = \frac{25{,}000}{275{,}000}$ = .0909

Substitute known values into the equation:

$$k_e = .12 + 0.0909\ (.12 - .06)$$

= .12545 or 12.55%

It's worth noting that this seems like a high value, and compared to the firm's ROIC or calculated *g* it is high and could lead to a troublesome valuations when applied to some model forms.

h. **Now, let's make a comparison of the firm's APV valuation with a valuation based on the KVD relation.**
To do this we're going to assume that $k_d = R_D$, $k_e = R_E$, and the marginal tax rate is also the average tax rate. Be aware that in some cases we have sufficient information to calculate the values of k_e, R_E, k_d and R_D such that we wouldn't necessarily hold $k_e = R_e$ and $k_d = R_D$.

Recall that the KVD relation is Value = PV_{DCF} + PV_{CV} where $CV_0 = \frac{NOPLAT_1\left(1-\frac{g_{2026}}{ROIC}\right)}{WACC-g}$.

First we'll need to calculate WACC given the values we know (this may not be a perfect WACC as we're using some calculated values that aren't expressly the same as those we might commonly use for WACC, but they're what we have available:

$$WACC = \left(\frac{E}{V} \times R_E\right) + \left(\frac{D}{V} \times R_D\right)(1-T_C)$$

$$= \left(\frac{200{,}000}{225{,}000}\ x\ .12545\right) + \left(\frac{25{,}0000}{225{,}000}\ x\ .06\right)(1-.35)$$

$$= .(8889\ x\ .12545) + (0.1111\ x\ .06)(.65)$$

$$= .1115 + .00433 = .11584 \text{ or } 11.58\%$$

Now let's calculate the PV_{DCF} value using our values for NOPLAT and our WACC as the discount rate:

Year	NOPLAT	PV_{DCF}	Total PV_{DCF}
2016	**16,200.00**	14,518.10	14,518.10
2017	**17,496.00**	14,051.68	28,569.78
2018	**18,895.68**	13,600.25	42,170.03
2019	**20,407.33**	13,163.32	55,333.35
2020	**22,039.92**	12,740.42	68,073.77
2021	**23,803.11**	12,331.12	80,404.89
2022	**25,707.36**	11,934.96	92,339.85
2023	**27,763.95**	11,551.53	103,891.38
2024	**29,985.07**	11,180.42	115,071.80
2025	**32,383.87**	10,821.23	125,893.02

So, PV_{DCF} = 125,893.02

We have the values for NOPLAT already and need to recall that $NOPLAT_1 = NOPLAT_{2026} = NOPLAT_{2025} \times (1 + \mathbf{g}_{2026+})$ = 32,383.87 x 1.04 = 33,679.22

Now calculate the $CV_{KVD} = \frac{NOPLAT_1\left(1-\frac{g_{2026}}{ROIC}\right)}{WACC-g} = \frac{33{,}679.22\left(1-\frac{.04}{.1533}\right)}{.11584-.04}$ = 328,195.78

Recall that this value is as of 2025, which is the 10th year of the DCF, so we need to discount it back to the present as follows: $PV_{CV} = \frac{CV_{KVD}}{(1+WACC)^t} = \frac{328{,}159.78}{1.11585^{10}}$ = 109,668.20

$VALUE_{KVD} = PV_{DCF} + PV_{CV}$ = 125,893.02 + 109,668.20 = 235,561.22

So how does this compare to $VALUE_{APV}$? $VALUE_{APV}$ = 235,561.93 so they're pretty similar and it's likely that aside from some possible rounding issues, they're virtually identical!

Okay, that worked out all too easily. Now we can discuss why ROIC = 15.333%. $VALUE_{APV}$ isn't necessarily expected to equal $VALUE_{KVD}$, they're very different valuation models and they each evaluate the value of the firm from very different perspectives. This last part of the ICP was intended to help you see that.

We can suppose that at some ROIC the two values would be the same though, so we can rework the equations such that $VALUE_{KVD} = VALUE_{APV}$ and solve for the ROIC at which the two values are equal.

This is going to get sort of complicated, but follow along and you'll see that it's just some simple substitution of values in the equation above.

$VALUE_{KVD} = PV_{DCF} + PV_{CV} = 125{,}893.02 + \frac{\left[\frac{33{,}679.22\left(1-\frac{.04}{ROIC}\right)}{.11585-.04}\right]}{1.11585^{10}}$

We already calculated $VALUE_{APV}$ = 235,561.93

So set the two equations equal to each other and solve for ROIC. Simple, right?

$125{,}893.02 + \frac{\left[\frac{33{,}679.22\left(1-\frac{.04}{ROIC}\right)}{.11584-.04}\right]}{1.11584^{10}}$ = 235,561.93

Subtract 125,893..02 from both sides

$\frac{\left[\frac{33679.22\left(1-\frac{.04}{ROIC}\right)}{.11584-.04}\right]}{1.11584^{10}}$ = 235,561.93 – 125,893.02 = 109,668.91

Multiply both sides by 1.11584^{10} = 2.9924

$\frac{33679.22\left(1-\frac{.04}{ROIC}\right)}{.11584-.04}$ = 109,668.91 x 2.9924 = 328,173.25

Multiply both sides by .11584 - .04 = 0.07585

$33{,}679.22\left(1-\frac{.04}{ROIC}\right)$ = 328,173.25 x 0.07585 = 24,891.91

Divide both sides by 33,679.22

$1-\frac{.04}{ROIC} = \frac{24{,}891.94}{33{,}679.22} = 0.7391$

Subtract 1 from each side

$-\frac{.04}{ROIC}$ = -0.2609

Multiply both sides by ROIC and divide both sides by -0.2609

$\frac{-.04}{-0.2609} = ROIC =$ .1533 or 15.33%..... *and now you know where I got the ROIC used at the front of the ICP.*